Force

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Introduction; In search of *Vis Visa*.   Prior to the time of Galileo (1546-1642 A.D.), science was conducted largely by philosophical argument. Questions concerning the workings of the natural world were posed, answers in the form of theory were supplied, and elegant arguments were also proposed to lend support to theory. In the Aristotelian worldview, experimentation was perceived to be "unnatural" and therefore not appropriate for discerning the workings of the natural world.

The cause of motion at this time in history is seen largely as due to physical contact, a "push" or a "pull".   The fact that a rock when released from some height would fall toward the earth was explained by stating "the rock simply went back to where it had come from, the earth". This event did not involve physical contact; And the observation that the moon was somehow linked to the earth, defied any explanation. The two phenomena were viewed as unrelated.

It may be interesting to know that documents of scientific content at this time in history were written exclusively in Latin. The phrase in use for the cause of motion was Vis Visa, the "living force" of motion. The earliest reference to the word "force" is recorded around 1300 AD. Its origin is from old French. It was used to describe "physical strength or vigor".

As we move forward to the time of Galileo (1546-1642 AD.), intellects of that day were still in search of the **cause** variable, the Vis Visa. Galileo however, brings a new tactic to the dispute of knowledge, the experiment. Galileo believes that experimental evidence, rather than verbal debate, will act as the better jury in the dispute of knowledge.

Galileo and other scientists of the day, for the purpose of discovering the cause of motion, conducted many experiments. It was thought that if this motion data could be understood, one might discover the Vis Visa, the cause for the motion.   Yet, in the days of Galileo, the experimental outcomes could only be considered peculiar. Humanity was in no position to fully understand the cause of motion, and could only classify the result.

[In the modern era, Galileo is thought of as the father of experiment. His cleverness in designing experiments, which verified premise, provided a new way of knowing. Experiment is now considered the gold standard for the verification of premise.]

Enter the era of Newton (1642-1727).   Of Newton's many attributes, the most powerful seems to be his ability to simplify situations and view them with a fresh eye. The mind was Newton's

primary laboratory and the mental experiment was Newton's method of knowing. Only after the mental experiment would lend reason to premise, would Newton seek verification in the physical laboratory. Newton was quite meticulous in his work. He was in no rush to publish his results until all avenues of dispute had been addressed and answered.

Newton was also in search of Vis Visa. Newton would succeed in providing mankind with the Vis Visa, or as he said, Vis Impressa. In1687 A.D., Newton published his thoughts of Force (the word "force" first used by Newton in 1686 AD). Newton's three force laws would provide a new understanding of force, acceleration, impulse, and momentum. Newton also provided the understanding of the force of gravitation. Newton purposed that the *reason* a rock released from some altitude would fall toward the earth, would not cease to exist at some abrupt altitude, but must extend far into space; to the moon and beyond. The conceptual tools needed to understand all of mechanics were put in place by Newton. The Calculus was also invented by Newton, providing the mathematical tool to manipulate these physical laws. Newton provided the tools that would ultimately open the pathway to the understanding of all of mechanics.

It is Newton's laws, which we will dissect in the paragraphs to follow. To follow his logic, it is useful to acquire the mindset of Newton at that time. We will attempt to tell this story as if we are watching Newton think.

Newton realized that the world is littered with many complex and interacting forces. It was this fact that complicated their understanding. Newton would require a simpler arena for his experiments. Newton would require an empty universe!

The First Law of Newton. Newton creates an empty universe in his mind. He now places an object in his hypothetical universe and then watches the object. Newton asks the question, " is the object in motion? Does the object have a velocity?" His answer is, of course. The object must have a velocity; the only question is "what is the value of the velocity; zero or perhaps non-zero." He concludes that the value of the velocity will depend on the frame of reference of the observation.

Newton's next question is "will the object change its velocity?" If the answer is yes, he reasons that a cause for that change in velocity must exist. Since his object is the only object in the universe (the all of everything), it would seem that no such cause exists. Thus Newton concludes, it must be true that objects, of themselves, will exist in a state of constant velocity – their natural state. An object, if in motion, will

remain in motion. An object, if at rest, will remain at rest. An object will persist at a constant velocity!

Newton now asks, "And what, if anything, could cause the velocity of the object to change?" Newton concludes that an intervention from outside of the universe would be required; from outside of the object.

This logic leads Newton to formalize his first law of motion; the law of Inertia (which means "law of constant velocity"). ***Objects persist in a state of constant velocity, unless acted upon by an external force.***

The Second Law of Newton. Newton was now in a position to reverse the logic of the first law. If an external force acts on an object, the object would then change its velocity. This statement is quite significant in that it establishes *Cause* and *Effect*. The cause - the Vis Visa - is an <u>external</u> Force, and the effect is a <u>change</u> in velocity. Since a change in velocity requires time, this change is in fact acceleration. The effect is acceleration. External force acting on an object leads to the object being accelerated.

Since both force and acceleration can be quantified, the next obvious question Newton will ask is "what is the mathematical relationship relating force and acceleration?" The answer to this question was obtained in the physical laboratory through experiment. Newton found that the relationship is proportional (linear). Doubling the force on an object will result in a doubling of the objects acceleration.

To obtain an equation, Newton must simply acquire a constant of proportionality. This constant has been given the name "mass". Newton's second law, in mathematical form, is:

$$\vec{F}_{external} = m\vec{a}$$

We can assign a physical meaning to the concept of mass: <u>Mass is the measure of an objects *resistance* to being accelerated.</u>

> This author wishes to emphasize here that mass is not weight, weight will be discussed later. Also, mass is not the amount of matter. Although that definition of mass is popular in areas of science such as chemistry, the amount of matter can be classified in numerous ways such as mass, weight, volume, or the count of the number of atoms. The chemist's agenda is ultimately concerned with the number of atoms or molecules, not with their resistance to being accelerated!

Since we have introduced a mathematical formula, we must also introduce a consistent set of units for each of three unit systems: the System International or SI system (formerly the mks), the cgs system, and the English system. These are presented in the following table:

Table I. Table of units

Unit System	Acceleration	Mass	Force
SI (mks)	m/s²	kg, kilogram	kg m/s² = Newton
cgs	cm/s²	g, gram	g cm/s²= dyne
Eng	ft/s²	slug	slug ft/s²= pound (Lb)

One can observe that history has introduced some complications into the simplicity of mathematics. The unit of force in the SI unit system is a "kg m/s²". However, history has also decided to honor Newton for his contributions to science. Thus, the unit of force in this system was re-defined to be a Newton. A "kg m/s²" _is_ a Newton, not by conversion but by decree! This fact will add to the memorization load of generations of students to follow. Newton would not likely have approved.

Similar re-definitions for various reasons also occur in other unit systems. A "g cm/s² _is_ a dyne. A "slug ft/s² _is_ a Lb, and this practice is not limited to force alone. The student of science must memorize these facts.

Another minor problem is that we must devise a way of bringing Newton's second law back from his hypothetical universe to our real universe. This is done through the magic of vectors. We have only to sum all of the real forces acting externally on our object as a vector sum, and that vector sum will represent an equivalent single external force acting on our object. Newton's second law becomes:

$$\sum \vec{F}_{external} = m\vec{a}$$

The above is a vector representation of Newton's second law. Application of this law is usually accomplished by splitting the one vector equation into two scalar equations; one for each (orthogonal) dimension.

$$\sum \vec{F}_{external} = m\vec{a}$$

$$\sum F_x = ma_x \qquad\qquad \sum F_y = ma_y$$

Where F_x and F_y are the rectangular components of each external force vector involved, and a_x and a_y are the rectangular components of the resulting acceleration vector.

At this point in our discussion, we know very little about force. Only that it is the cause of acceleration. Newton believed that forces could arise by other than physical contact. He believed that "Action at a

distance" could also be considered a force. The attraction of an object toward the earth for example, would constitute a real force. And if an object is attracted toward the earth, is the earth not also attracted toward the object?

Newton was aware of the relationship between the ocean tides and the position of the moon. This was evidence that the moon pulled on the earth. It should also be the case that the earth pulled back on the moon. And when a person pushes an object, it is also true that the object pushes back on the person. Newton realized that forces are created in pairs, by various types of interactions between objects.

The Third Law of Newton. It is the third law that will provide much insight into the concept of force. Newton's third law is a law of interaction between objects. (We notice that Newton has now placed a second object in his hypothetical universe.) It states:

Given the discovery of a force acting on an object, there is a second force to be discovered. This second force being:
1. **Equal in magnitude to the first force.**
2. **Opposite the direction of the first force.**
3. **Of the same category of interaction as the first force.**
4. **Acting on the <u>other</u> object involved in the interaction.**

Newton's third law states that forces arise as a result of various types of interactions between objects. Two objects are required. In a practical sense, the third law gives all the necessary clues to find the "second" force when one has found the first.

Categories of Force Interactions. Most textbooks for beginning physics will list four categories of force. These are:
- Gravitational Force.
- Electro-Magnetic Force
- Strong Forces.
- Weak Forces.

of which the last two categories, Strong and Weak Forces are believed to be found only in the nucleus of an atom. It is maintained by the scientific community today that any and all real forces will fall into one of the above four force categories. There is no fifth category.

While this author is in support of the above, in the study of Newtonian Mechanics, one rarely discusses the inner workings of the atom, put aside the nucleus of an atom. Newtonian Mechanics is concerned with a much more "macro view" of the workings of the world. It is for this reason that this author will suggest a more applicable list of

force categories. In the macro view of nature, the four force categories of concern are:

1. Gravitational Forces.
2. Elastic Forces.
3. Friction Forces.
4. Electro-Magnetic Forces.

In this list, we have removed the forces within the nucleus, and added the categories, elastic and frictional forces. These two force categories are the "touch" forces. In the next few paragraphs, we will add detail to each of the force categories above.

Gravitational Force. Newton will discovered the force of gravity. Any two objects will be attracted toward each other by a gravitational interaction. The magnitude of this attraction depending on the product of each objects mass, and the inverse of the distance between the objects squared. In the language of mathematics,

$$F_{Gravitation} = G\frac{m_1 m_2}{d^2}$$

where G is considered the universal gravitational constant. In SI, the constant has the value, G=6.67 x 10⁻¹¹ Nm²/kg².

Why the force of gravitation exists is not understood. Even so, we may still predict its value based on the formula above. Notice that if we compute the value of the gravitational attraction between two adults of mass 70 kg, standing 1 meter apart, and the value arrived at by the above formula would be 3.27 x 10⁻⁷ N. (.000000073 Lbs). Considering its small magnitude, it is quite understandable why we have not felt this force of gravitation.

Now consider applying the above formula to an object of mass "m" on the planet's surface. The distance of separation is now the radius of the earth and the remaining mass is the mass of the earth. The formula is:

$$F_{Gravitation} = \left(G\frac{M_{earth}}{R_{earth}^2}\right)m$$

Here, we have collected in parenthesis, all quantities which are <u>constant</u> for the computation of an object on the planet's surface. We evaluate the parenthesis first as:

$$\left(G\frac{M_{earth}}{R_{earth}^2}\right) = (6.67 \text{ x } 10^{-11} \text{ Nm}^2/\text{kg}^2)\frac{(5.98 \text{ x } 10^{24} \text{ kg})}{(6.37 \text{ x } 10^6 \text{ m})^2} = 9.83 \text{ m/s}^2$$

This value is clearly recognized by all beginning students of physics as the acceleration due to the force of gravity, "g". Thus, near the earth's surface, we may replace the above computation by a simplified version for this special case,

$$F_{gravitation\ near\ earth} = mg$$

We now define the term "weight" as the force of gravity, acting on objects near the earth's surface.

$$W = mg$$

The difference between mass and weight should now be clear. Weight is a force interaction between the earth and an object on the earth's surface. Weight is the force of gravity, acting on an object near the earth's surface. Mass is not a force. Mass requires no interaction. Mass is not a vector quantity; it is scalar. Mass is a measure of an object's resistance to being accelerated.

Electro-Magnetic Forces. Although it is not the intent of this author to discuss the Electro-Magnetic forces in detail, it is however useful to point out some aspects of the *electrostatic force* since this force does indirectly have relevance to discussions that follow. The topic of Electro-Magnetic forces is not directly related to Newtonian mechanics, and is explored in subsequent courses in electricity and magnetism.

The electrostatic force is in many ways similar to the gravitational force in that its magnitude is inversely proportional to the square of the separation distance. The electrostatic force however does not depend on the product of the masses of the two objects, but on the product of the two objects *charges*. In equation form,

$$F_{electrostatic} = k\frac{q_1 q_2}{d^2}$$

where "q" is the charge on each object in Coulombs, and "k" is the universal electrical constant ($k = 9.0 \times 10^9 \ Nm^2/C^2$). The relationship is known as Coulomb's Law.

Electrical charge exists in two varieties. These varieties are called "positive" and "negative". When both charges are dissimilar, the direction of the electrostatic force is attractive (similar to gravitation), but when the charges are similar, the direction of the force causes the charges to repel each other.

Within the atom, we know that the electrons moving in the outer 'shells' are negatively charged. The protons within the nucleus core are positively charged. Both types of particles have mass although the gravitational attraction between the particles is very small when compared to the electrostatic attraction. Electrostatic forces are the "glue" of the atom, as well as the bonding force between collections of atoms to form molecules.

Elastic Forces. There is a material test often performed in engineering, to lend insight into the behavior of that material under load. The Tensile test. In this test, a sample of the material is pulled under tension while various measurements are collected. Two quantities are computed from the measurements.

They are:

$$\text{Normal Stress} = \frac{F_{\text{of pull}}}{A_0} \quad \text{And} \quad \text{Normal Strain} = \frac{\Delta l}{l_0}$$

where A_0 is the original cross sectional area, l_0 is the original length; F is the pulling force, and Δl is the resulting change in length.
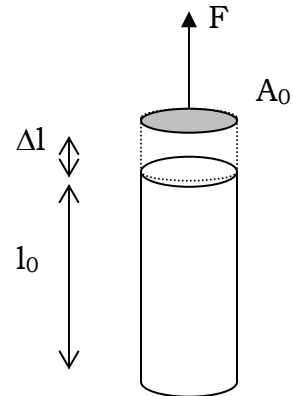
The stress as a function of strain is then plotted on a graph. The resulting graph, for a sample of mild steel would look similar to the figure below.

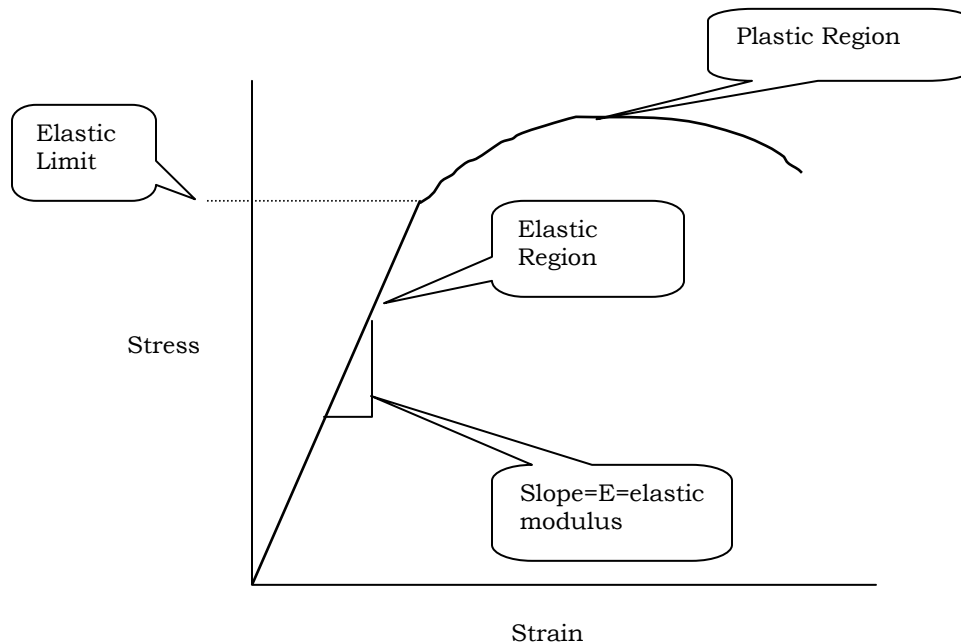Figure 1. Dimensions measured in Tensile test.

Figure 2. Stress vs. Strain diagram for mild steel

The stress strain relationship is linear in the elastic region of the test. While loading in the elastic region, no permanent deformation is done. Permanent deformation requires that the stress exceed the elastic limit.

The engineer will compute the slope of the line in the elastic region. This slope, "E", is given the name, Elastic Modulus (or Young's modulus). With the elastic modulus, the engineer can determine the mathematical relationship between stress and strain. It is,

$$\text{Stress} = E * \text{Strain}$$

Substitution of the fundamental measurements in the above gives,

$$\frac{F}{A_0} = E\frac{\Delta l}{l_0}$$

and on rearrangement, collecting constants in the parenthesis, we obtain,

$$F = \left(\frac{EA_0}{l_0}\right)\Delta l$$

Here it is easily seen that the elastic force, F, is directly proportional to the amount of deformation, Δl. The elastic force is perpendicular or normal to the surfaces of contact, and in an amount proportional to the deformation of the objects.

In physics and engineering, it is often the case that a "spring" is used as a model for an elastically deformable element. The equation relating the spring's deformation to the resulting elastic force is known as Hooke's Law. In mathematical terms, Hooke's Law is:

$$\vec{F}_{Spring} = -k\vec{x}$$

where "k" is the "spring constant", measured by experiment. The value of "x" is the deformation of the spring (x is assumed to be zero when the spring is undistorted). The minus sign appears since the force applied by the spring to its surroundings is opposite the direction of the distortion.

As to the source of the elastic force, we have only to imagine a tensile test specimen one atom in diameter. In pulling this specimen, we seek to displace its atoms from their equilibrium positions. These positions are determined by a delicate balance of the attractive and repulsive electrostatic forces between the atoms within the material. The elastic force, in essence, belongs to the family of electro-magnetic interactions. It is however useful to consider this force in a more macro manner.

Two common terms are used to describe the elastic force: Tension, and Compression. We define tension as the force with which a member pulls on its supports. We define compression as the force with which a member pushes on its supports.

If a rope is in tension, then it is pulling on its supports. We can directly measure the tension by placing a spring scale in the rope at any point and reading the force value displayed on that scale. The spring

scale in a sense becomes one of the ropes supports. It should be noted that the rope will pull on its support with the same force that the support pulls back on the rope as explained by Newton's third law.

Friction Forces. Most people believe that any force, which slows an object, is a friction force. Indeed, the label friction is mistakenly placed on phenomena that have little to do with friction. "Air friction", for example, is for the most part an elastic compression between an object moving through the air and the air molecules them selves; it is an elastic force.

Like the elastic force, friction is also a touch force. The direction of the friction force is however parallel to the plane of the touch. The term *shear* is used when the force is within the plane of the touch. Friction is caused by the resistance to shearing the surface molecules, which have bonded at the surface interface. Thus the friction force, in essence, belongs to the family of electro-magnetic interactions.

When trying to predict the magnitude and direction of the friction force, two distinctly different situations arise: surfaces in relative slippage, and surfaces without slippage. We will analyze the case of slippage first.

Slippage.

The direction of the friction force, within the plane of the touch, is easily determined if there is relative slippage between the surfaces. With relative slippage, one may select one surface as the frame of reference and observe the other surface's velocity from that frame of reference. <u>For relative slippage, the force of friction on a surface is in the direction opposite to that surfaces velocity, measured from the other surfaces frame of reference.</u> When slippage occurs, the prefix "kinetic" friction is used.

A number of factors may, or may not, contribute to the magnitude of the force of friction between two materials, depending on the relative compliance of the surface pair and the compression at the surface. This list includes:
1. Texture.
2. Surface Area.
3. Compression at the surface.
4. Material makeup.
5. Relative Velocity.
6. Temperature.

When a surface pair has a gross regular texture, the mechanism preventing slippage is mechanical linkup. This linkup is similar to two engaged gears. For slippage to occur one of the gears must fail by shear;

the external forces will have reached a level such that they exceed the shear strength of the material. Both a large shearing force, and a large compression at the surfaces induce the shearing of a material pair. When the shear strength of the weaker material is exceeded, shear and subsequent slip will occur. The word "friction" is not used to describe the above condition.

As the surface pair becomes more refined and random in texture, failure and subsequent slip by exceeding the shear strength of the material can also occur if the surface compression is large enough. This type of failure is also common when one of the two materials has compliance, which is significantly larger that the other material involved.

A car tire, for example, is much more compliant than the pavement it is in contact with. The compression force between the tire and pavement, due to the weight of the car, is also quite large. This compression forces the softer material of the tire into the irregularities of the pavement and as the car accelerates, the shear strength of the tire is exceeded as evidenced by the black stripe on the roadway. Although this mechanism is often called "friction", the term is loosely applied and would better be named as simply shear.

It is when the loading compression at the surfaces is light to moderate, and the compliance of the materials is more similar, that we properly call the mechanism friction. Under these conditions, and <u>with surfaces of a more refined and random texture, we find that the actual texture has little to do with the magnitude of the frictional force</u>.

Surface Area, or area of contact, is also believed to be a factor determining the magnitude of the frictional force between two surfaces. Much of our reasoning again comes from the car tire. And while it is true that under the conditions that the car tire is subjected to, increasing area also increases this "friction", when the compression force on the car tire is decreased, or the area of the tire contact is significantly increased, there becomes a point where the area has little effect on the magnitude of the "frictional" force. For hard, non-deformable surfaces, the area has NO effect on the magnitude of the friction force. This can easily be demonstrated by experiment.

It should be pointed out that the use of the word *area* in the above discussion refers to the area as measured by a ruler or what we will call the *macro area.* With hard surfaces, the macro area is NOT the true area of contact. When the surface pair is examined under microscope, it is seen that even the smoothest macro surface exhibits a texture resembling the landscape of a mountain range. When these two surfaces are put in contact, the true area of contact is significantly less than the macro area.

As was pointed out in previous paragraphs, friction is caused by the resistance to shearing the surface molecules that have bonded at

each surface. Thus, a deeper understanding of the mechanism of friction requires an understanding of a solid-solid chemical reaction between reactants, and the probability that these reactions will indeed form a product, which will resist shear. Consider two molecules, one at the surface of a stationary object (molecule "A"), and the other at the mating surface of an object in motion relative to the stationary object (molecule "B"). When molecule B is far from molecule A, the electrostatic forces between the two molecules are extremely small. As molecule B approaches molecule A, the electrostatic force between them increases. Since molecule B is in motion, relative to molecule A, molecule B will only be near molecule A for a short period of time. If these two molecules are to bond and form a chemical product, it is not enough to simply share an electrostatic force in the usual manner, forming a bond also requires that a number other complicated conditions be resolved between the two molecules. These conditions include geometrical alignment (docking), the promotion of electrons (hybridization), and other quantum mechanical considerations. These considerations require time to complete. If the required conditions can be completed in the time available, a bond is formed. If they cannot be completed while the two molecules are in close proximity, no bond is formed and those two molecules do not lend themselves to the overall friction force. The completion of a reaction and the subsequent friction force is then a matter of probability. Given any 100 molecules in close proximity, only a random few (about 50%) will result in the creation of a bond and lend to the friction force.

Compression at the surface and material makeup are two factors that play an important role in the magnitude. With increased compression, the objects deform elastically. This brings more molecules into the proximity of each other and thus increases the friction force. The type of materials involved determines the effective elastic modulus of the system, thus controlling the number of new molecules brought together under compression, and the materials also determine the strength of the product formed.

Increasing the relative velocity between the two surfaces has the effect of reducing the time available to complete a bond and thus reduces the magnitude of the friction. The effect of increased velocity is however a minor effect at slow to moderate speeds in most cases and is usually ignored accept in one important case; Zero relative velocity. When there is no slippage between the material pairs, the time is sufficient to complete 100% of the bonds between those molecules that are in proximity. The force required to then initiate slippage is significantly increased.

Increasing the temperature of the materials tends to increase the rate at which the bonding conditions will occur (the rate of the reaction), thus increasing the friction force. The effect of increasing temperature by a narrow amount usually produces only a minor increase in friction and is usually ignored.

Physics and engineering will usually "model" friction with a reasonably simple formula. It is:

$$F_{friction} = \mu_k C$$

Where μ_k is the coefficient of kinetic friction, and C is the compression force acting perpendicular to the surface pair. The coefficient of friction is an empirically measured value and is classified based on the material pair involved; values of which are less than one. This model of friction works very well since its origin lies in experiment.

Non-slippage. A more complicated situation arises when there is no slippage between the two surfaces. Since the velocity rule does not apply, one must carefully examine all other existing forces on that object first. For the case of no slippage, the force of friction will be in whatever direction, and with whatever magnitude, necessary to make the observed effect consistent with Newton's second law.

It is often desirable to predict the amount of friction force, which one must just exceed, in order to initiate slippage. To accomplish this, a model similar to the kinetic friction model is employed. It is:

$$F_{friction\,MAX} = \mu_s C$$

Where μ_s is called the coefficient of static friction. This author finds the word "*static*" a poor choice and prefers the word "*starting*". The coefficient of starting friction is also measured from experiment and classified by material pair.

It should be emphasized that in a non-slippage condition, the magnitude of friction may be any value from zero, up to and including F_f MAX. If the surfaces are not slipping, and if it is also known that the surfaces are not on the edge of slipping, the ONLY way to predict the amount of friction is with Newton's 2nd Law.

Considerations in Preparation to Applying Newton's Laws. All applications of Newton's laws must start with the construction of a sketch of the objects involved, and a force diagram. Since forces are vector quantities, they may be represented by arrows. This technique of representing a vector with an arrow should be familiar since it is commonly used in the study of motion. We will however add an extra component to the force vector arrow; A tail.

Figure 3. Illustration of a force vector, showing addition of a tail.

The tail is added to the force vector because in addition to representing the magnitude and direction of the force, we wish to convey the location at which the force acts on an object. The tail is placed at that location.

When applying Newton's laws to the center of mass of an object, the exact location of the force on the sketch is not important. The location is assumed as the center of mass. When applying Newton's laws to an object that has its mass distributed in space however, the precise location of the force is extremely important. In this case, the force locations may lead the engineer to also consider a Torque analysis and this Torque analysis requires that the forces be located precisely.

A second consideration is the selection of a coordinate system. The choice of coordinate system belongs to the toolbox of the problem solver, not an author, not the math department. Although it is true that a given problem, if solvable, can be solved with any orthogonal coordinate system, a smart choice of coordinate system may often lend itself to a less complicated solution.

Consider a situation where the problem solver has determined that there is a constant acceleration involved. We compare two choices of coordinate system as shown below. With the choice on the right, the acceleration will have both a non-zero "x" component, and a non-zero "y" component. With the choice on the left, the "y" component of the acceleration will be zero and all the acceleration appears in the "x" dimension. The second choice will produce equations that are much easier to solve.

$$\sum F_x = ma$$
$$\sum F_y = 0$$

$$\sum F_x = ma_x$$
$$\sum F_y = ma_y$$
$$|a| = \sqrt{a_x^2 + a_y^2}$$
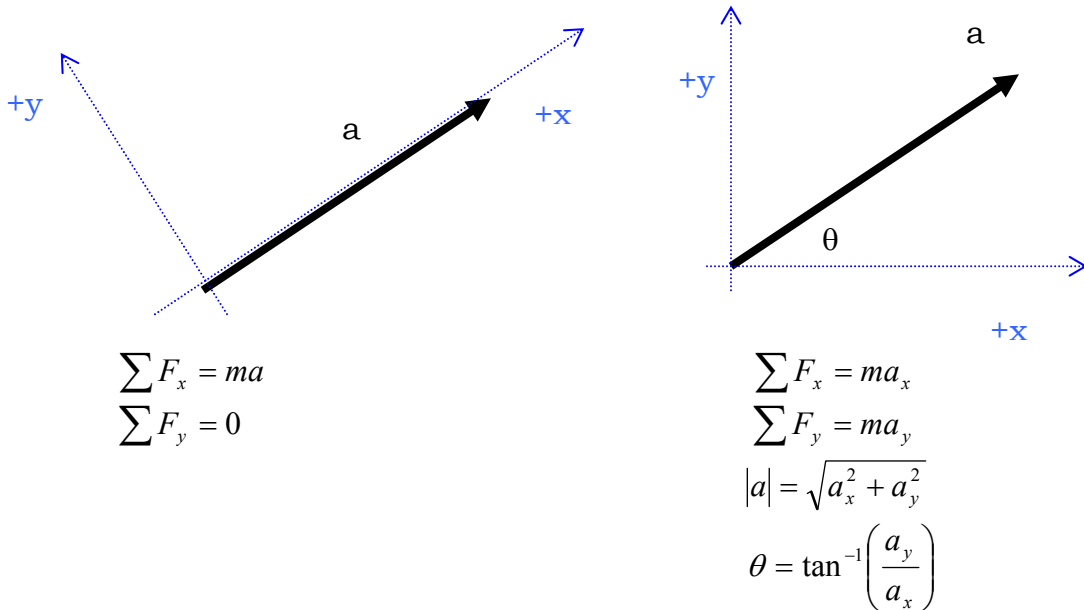$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right)$$

Figure 4. Illustration of choice of coordinate system.

Our final consideration involves the "y" equation on the left diagram. If the acceleration in that dimension is zero (or if the object is mass less), then the sum of the external forces in that dimension result in zero. The phrase, "forces in balance" is often used to describe this condition. The phrase can be interpreted as: *the forces in the "+y" direction are equivalent to the forces in the "-y" direction.* This can be shown as follows:
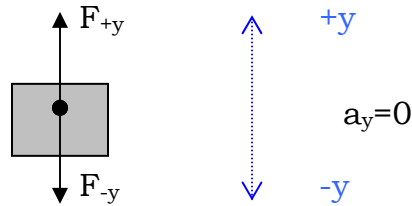

Figure 5. Illustration of involved forces in a force balance.

$$\sum F_y = 0$$

We separate the forces in the general sum into plus and minus forces.

$$\sum \left(F_{+y} + F_{-y} \right) = 0$$

Next, we change the order of addition.

$$\sum F_{+y} + \sum F_{-y} = 0$$

Next, we factor the minus sign from all negative forces in the second sum and add the absolute value symbols for the sake of clarity.

$$\left| \sum F_{+y} \right| - \left| \sum F_{-y} \right| = 0$$

The second sum is now moved to the other side of the equation to produce the force balance.

$$\left| \sum F_{+y} \right| = \left| \sum F_{-y} \right|$$

Experience shows that the force balance format is a more efficient way to author the second law when the acceleration is zero since the algebra in the above steps is bypassed.

General Application Procedure. Many beginning students of physics approach the solving of physics problems in the same manner as they used in their beginning mathematics classes. That is, "show me how to do one of that category, then let me do 100 more, and I will understand". This way of learning does tend to create an excellent problem solving technician, however when the category of problem is different, the student requires another trip to "show me" school. The understanding part of the solution, the application of the rules and the strategy of their application, is overlooked. True problem solving, is the acquisition of a solution to a problem, which has not previously been solved. This is accomplished by starting with a clear understanding of the goal or quantity to be found, and the rules, which relate that goal or quantity to other known factors. Once these relationships are

determined, interrelationships are meticulously explored for a common thread; always keeping an eye on the target goal. The process is often described as "good ole fashion detective work". And it should be pointed out that not all problems would have solutions with the information or relationships available. Still, if no attempt is made, the outcome is sure.

Most problems requiring a force analysis, can be set up by following the procedure outlined below:

1. Select an Object. Draw a sketch.
2. Find all forces that: a) Act on the object. b) Are External. Use your knowledge of forces, not your memory.
3. Draw a Force Vector Diagram.
4. Do the lines of action of all forces intersect at a single point in space? If No, then Torques should also be considered.
5. Assess the object's acceleration.
6. Select a coordinate system.
7. Replace any force not in the direction of a coordinate axis with rectangular components, which are.
8. For each dimension, assemble Newton's second law.
9. Identify any other pertinent force or motion relationships.

At this point, the physics part of the problem solution is suspended, and the problem becomes a math problem.

Application Examples.

Problem: A block of unknown mass is placed on a 30-degree incline and released. $\mu_k = .20$ for this material pair. The block is observed to slide down the incline. What is the acceleration of this block?

Solution: We start by asking the question "what kind of problem is it?" The goal is to determine acceleration thus; we start by considering it as a motion problem. Motion problems, in general, ask about position, velocity, acceleration, and time. In reviewing the information supplied, we see that the information supplied is not sufficient to attain a solution as a motion problem. The problem is now considered as a force problem. Force problems, in general, ask about forces, accelerations, and other constants associated with force.

We select an object. The object will be the block since we wish to know about its acceleration.

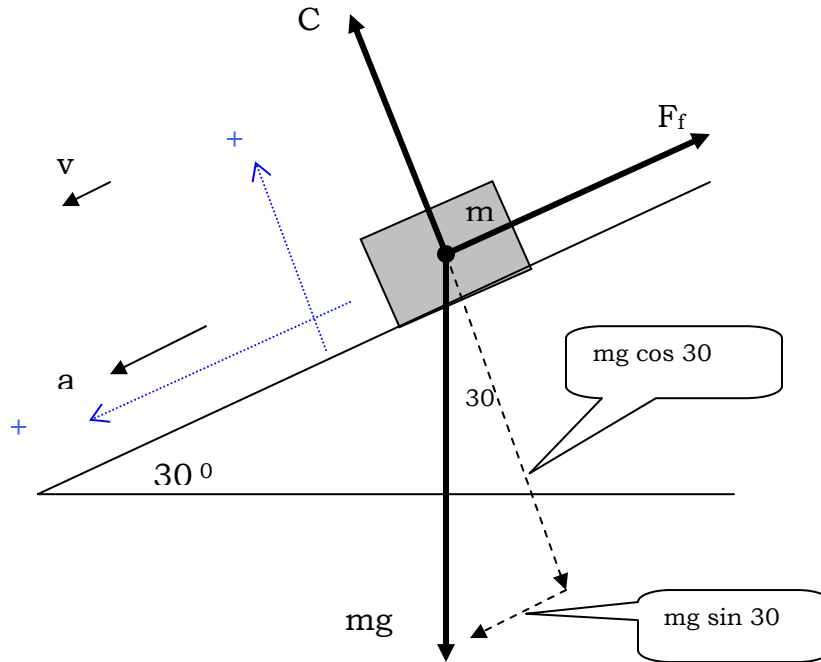We construct a sketch containing all given information.

Figure 6. Sketch of problem, complete with a vector diagram and coordinate system.

Forces are identified and the force vector diagram constructed by going down the checklist for forces. Since the object is near the earth, it will have a weight = "mg" pointing toward the center of the earth. The object is touching the incline so there will be an elastic force. We expect no significant deformation of the incline and object, thus this touch will not be modeled with a spring. We do however include the elastic force as a compression force acting on the object. The touch will also produce a friction force. With our frame of reference as the incline, we see the velocity of the block as down hill and thus assign the direction of friction on the block as up hill. There are no relevant electro-magnetic forces involved. All forces are assumed to act on the center of mass since we have no geometry given for the block.

It is not expected that the block will leave the incline's surface so the acceleration perpendicular to the incline must be zero. Acceleration parallel to the incline will occur, and it is the desired goal. For these reasons, we select the coordinate system as perpendicular and parallel to the incline; positive in the directions shown.

The force "mg" is now replaced with two rectangular components in the dimensions of the chosen coordinate system: (mg cos 30) and (mg sin 30).

We now author two statements of Newton's second law, one for each dimension, with the aid of the force vector diagram constructed.

Also included in the equation list is the application of the force of friction model.

$$|C| = |mg\cos 30|$$

Friction Model

$$F_f = \mu_k C$$

System: Object m, parallel dimension, a

$$+|mg\sin 30| - |F_f| = ma$$

Notice that the equation in the perpendicular dimension is a "force balance" since the acceleration in this dimension is zero. A force balance can be easily recognized by observing the equation header contains "a=0". The equation in the parallel dimension is a $\sum F = ma$. This can also be recognized from the contents of the equation header, "a". Also, notice the liberal use of the absolute value symbols. Most textbooks do not include them however we have done so here to make it clear that we are working with the magnitudes of these forces. The direction of these forces is known, and thus their signs are known through observation of the chosen coordinate system.

We have now only to solve the system of equations for the target goal, "a". In this case, we may complete the solution by substitution as follows:

System: Object m, perpendicular dimension, a=0

$$|C| = |mg\cos 30|$$

Friction Model

$$F_f = \mu_k C$$

System: Object m, parallel dimension, a

$$+|mg\sin 30| - |F_f| = ma$$

Substitution as diagramed above, leads to:
$$\left|mg\sin 30\right| - \mu_k\left|mg\cos 30\right| = ma$$
Notice that the mass of the block will cancel in this problem. We now solve for the acceleration to arrive at:

$$a = \left|g\right|\left(\sin 30 - \mu_k\cos 30\right)$$
$$a = (9.8m/s^2)(.5 - (.2)*(.866))$$
$$a = +3.2 \text{ m/s}^2$$

The last step in any problem is to review the result. Positive acceleration indicates that the acceleration is in the direction chosen positive in the coordinate system, down the incline in our problem.

Problem: It is determined through experiment that when a car is driven around a 400 ft diameter circle, the maximum speed at which the car can complete the maneuver with out sliding of the circle is 52 mph. Use this data to determine the effective coefficient of starting friction between the car and pavement for this maneuver.

Solution: First, we identify the problem as a force problem since our goal, the coefficient of starting friction, is linked to a force relationship through the friction force model equation. We also note that the surfaces are not hard, non-deformable surfaces, thus we are pushing the limits of reality of this model equation. Still, the data is collected for "this" car under "these" conditions, and the representation of the coefficient of starting friction will be valid for "this" car, pavement, and tire combination.
We select the car's center of mass as the object and draw a sketch.
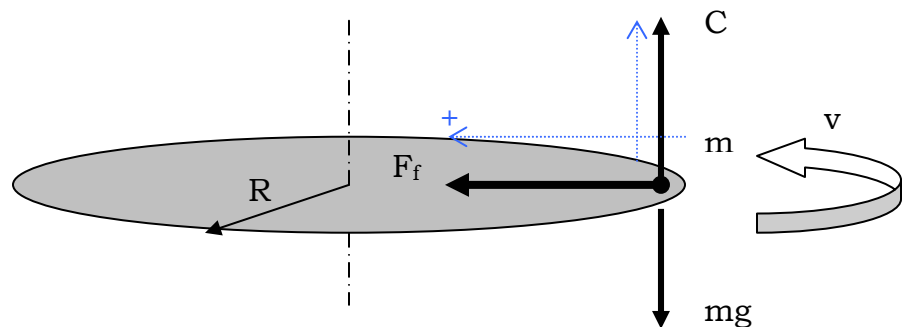
Using our list of forces, the car is pulled toward the earth by the force of gravity, mg. There is a compression force up on the car from the pavement, C. Neither of these two forces explain why the car is moving in a circular path. We know from motion studies that an object moving

in a circular path will have a radial inward (centripetal) acceleration, which is caused by force. We conclude that the force of friction between the car and pavement must act radial inward on the car, F_f. No other forces act on the car. Since we are on the edge of slippage, the appropriate friction model will be $F_{f,MAX} = \mu_s C$.

Many people believe that there is also a radial outward force on the car. A centrifugal force. This outward centrifugal force _does not exist!_ There is no object interacting with the car to produce an outward force on the car. Centrifugal force is a fictitious force. It does not exist.

Our force diagram is now complete.

The car is not accelerated in the vertical dimension. The car is accelerated in the radial dimension. We will select the coordinate system vertical and radial. This is now included in our sketch.

No forces need be replaced by rectangular components.

We assemble Newton's law for each dimension. The radial equation will be an "F=ma" since this is an accelerated dimension. We will use a "force balance" in the non-accelerated vertical dimension.

<div align="center">

System: Object m, vertical dimension, a=0
$$|C| = |mg|$$

System: Object m, radial dimension, a_c
$$|F_f| = ma_c$$

</div>

We also identify the other pertinent relationships. These are the motion relationship, $a_c = \dfrac{v^2}{R}$, and the friction model, $F_{f,MAX} = \mu_s C$.

Our equation set is seen below. The target goal for our solution, μ_s, if found in the friction model equation. We substitute into that equation as diagramed.

System: Object m, vertical dimension, a=0   Friction model:

$$|C| = |mg| \qquad\qquad F_{f,MAX} = \mu_s C$$

System: Object m, radial dimension, a_c Motion relationship:

$$|F_f| = ma_c \qquad\qquad a_c = \frac{v^2}{R}$$

The resulting substitutions give the composite equation,

$$m\frac{v^2}{R} = \mu_s mg$$

Canceling the mass of the car, and solving for the coefficient we get,

$$\mu_s = \frac{v^2}{Rg}$$

Preparing for a numerical solution, we note that the given velocity must be converted to a value in feet/sec to conform to the standard English system units.

$$52 \text{ mph} = 76.3 \text{ f/s}$$

$$\mu_s = \frac{(76.3 \text{ f/s})^2}{(200 \text{ ft})(32 \text{ f/s}^2)} = .909$$

All units cancel in the above computation (as they should) and the answer is reasonable.

In summary of this solution, it should be noted that the application of Newton's second law to a circular motion problem is identical to that of a non-circular motion problem. The only unique addition is the motion relationship describing the radial acceleration.

Problem: A 500 gram mass is placed on a level table. This mass is connected to a 200 gram mass by means of a "mass less cord" which passes over a "mass less, frictionless" pulley as shown. The coefficient of kinetic friction between the 500 gram mass and the table is (.1). The system is released from rest and observed to accelerate. Find the acceleration of the system and the tension in the connecting cord.
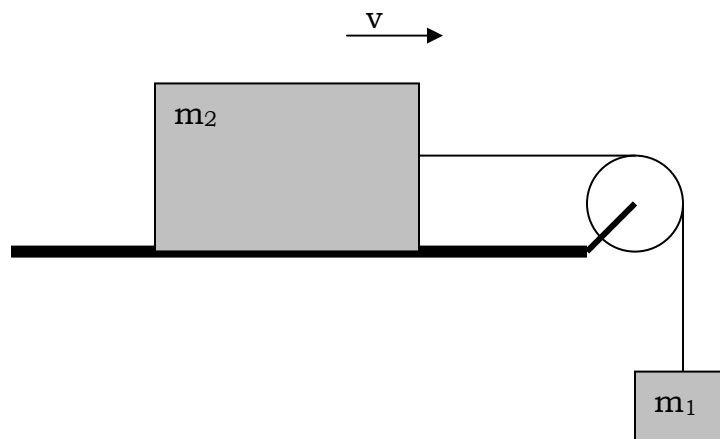
Solution: We note that there are two objects. Both will accelerate. We also note that there is little motion data (distances, times, velocities) so the solution will be attempted as a force problem. The function of a mass less and frictionless pulley is to change the direction of the cord; It cannot change the tension in the cord. We select both masses and proceed with the force analysis.

The force of gravity will act on each object; m_1g, and m_2g.

There is an elastic compression between m_2 and the table acting up on m_2, C. The cord will have an elastic tension. This tension will pull on m_1 and m_2 which are the cords supports; T.

There will be a friction force acting on m_2 in the direction opposite of its velocity (measured from the table); F_f. We now complete the force vector diagram on both objects center of mass as seen below.
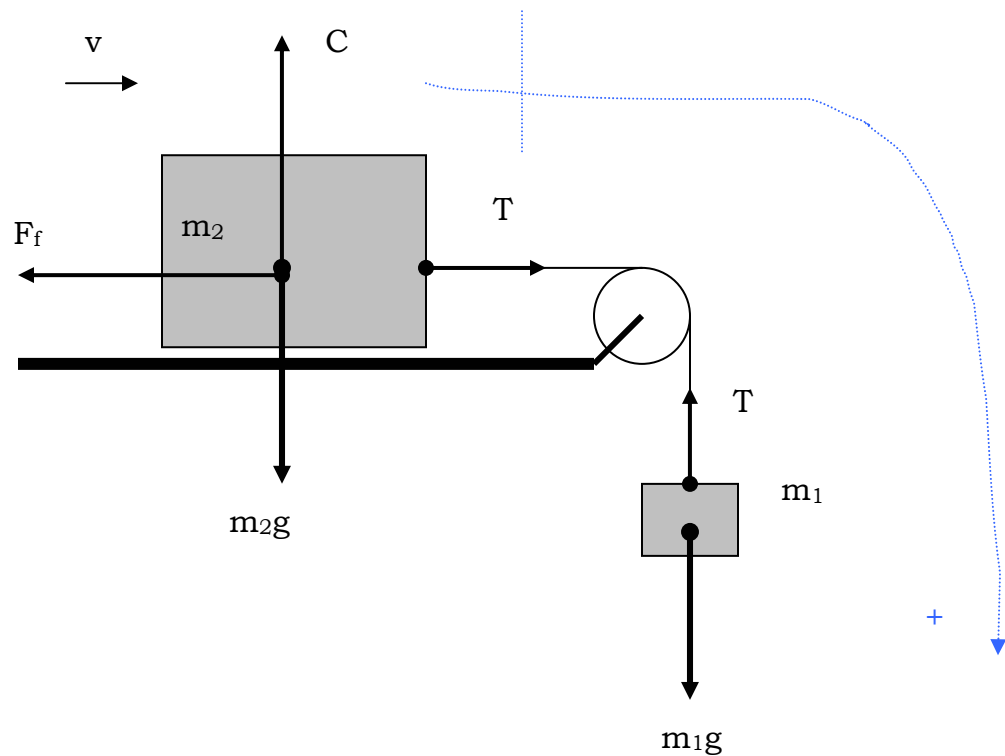
M_2 will have zero vertical acceleration, and non-zero horizontal acceleration. M_1 will have the same acceleration magnitude as m_2; and if we are clever, the same sign. We choose to the right (for m_2) and down (for m_1) as positive and indicate the coordinate system on the diagram above at this time.

No forces need be replaced by rectangular components.

We assemble a vertical force balance for m_2, and two F=ma's for each mass.

System: m_2, vertical dimension, a=0   System: m_2, horizontal dimension, a

$$|C| = |m_2 g| \qquad\qquad\qquad\qquad |T| - |F_f| = m_2 a$$

<u>System: m₁, vertical dimension, a</u>
$$|m_1 g| - |T| = m_1 a$$

Note that the signs (+ or -) in the "F=ma" equations are consistent with those of the coordinate system directions.

Finally, we include the kinetic friction model equation to complete the equation set seen below.

<u>System: m₂, vertical dimension, a=0</u>
$$|C| = |m_2 g|$$

Friction model:
$$F_f = \mu_k C$$

<u>System: m₂, horizontal dimension, a</u>
$$|T| - |F_f| = m_2 a$$

<u>System: m₁, vertical dimension, a</u>
$$|m_1 g| - |T| = m_1 a$$

To solve, we select "a" as our target variable. The variable appears in both equations on the right. The addition of those two equations will eliminate "T". We do so to give:

$$|m_1 g| - |F_f| = m_1 a + m_2 a$$
$$|m_1 g| - |F_f| = (m_1 + m_2) a$$

We now complete the substitutions for the force of friction giving,

$$|m_1 g| - |\mu_k m_2 g| = (m_1 + m_2) a$$

And we solve for the acceleration,

$$a = g\left(\frac{m_1 - \mu_k m_2}{m_1 + m_2} \right)$$

$$a = (9.8 m/s^2)\left(\frac{.20 \text{ kg} - .10(.50 \text{ kg})}{.20 \text{ kg} + .50 \text{ kg}} \right) = 2.1 \ m/s^2$$

To compute the tension in the cord, we may use either of the two equations from "F=ma". We choose,

$$|m_1 g| - |T| = m_1 a$$

and solve for "T" to get,

$$|T| = m_1 (|g| - a) = .20 \text{ kg}\left((9.8 \text{ m/s}^2) - 2.1 \text{ m/s}^2\right) = 1.5 \text{ kgm/s}^2 = 1.5 \text{ N}$$

Final Thoughts. With the outlined procedure, we have separated the physics part of the solution from the mathematical part of the solution. In the physics segment, at no time did we encounter the step – *worry about how we will solve this problem!* There are many concrete steps to perform before "worry" becomes a factor, if at all. All of these steps must be addressed before we are in a position to consider the problem as non-solvable.

Some problems may contain multiple objects. It may be necessary to execute the procedure for each object. Here, Newton's third law will take on more importance. When a force is identified, the problem solver should also identify the "second" force involved in that interaction and the object that it act's on.

Multiple objects may lead to multiple equations in multiple unknowns. Under these conditions, organizational skills including equation labeling, artistic layout, and a carefully constructed vector diagram (with coordinate systems) will help the problem solver both construct the necessary equations as well as solve them mathematically.

Finally, meticulous organization will allow the problem solver to communicate the solution to the intended person. A solution should always be viewed as a *presentation*; to yourself, your instructor, or your employer.